Calutta University
Semester: 4
session: January 2020 to June 2020
Paper: CC9 Unit:2

Hypothesis Testing About The Difference Between Two Dependent (Correlated) Means

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Concept of dependent sample t test

- The dependent samples t test (sometimes called the paired samples t test) is used in a -
- **Single group method**: It consists of repetition of a test to a group. Initially a baseline measure is taken from all the individuals in a group (serves as control group) , then the desired experimental treatment is given, followed by the re-administration of the same test to the same individuals of the group (serves as experimental group). The two sets of scores form paired observations and these two scores are correlated.

  For example:

  1. The achievement test scores of 10 individuals before and after practice
  2. The change in the strength of knee jerk reflex after injecting adrenaline.
- **Equivalent Group method**: Two separate equivalent groups are taken for experimental study. Equivalent groups can be formed in the following ways:

  Matching pair technique-Matching is done initially by pairs so that each individual in the first group has its equivalent or a match in the second group in terms of some variable which are going to affect the results of the study like age, sex, intelligence interest, aptitudes etc.
- *Matching group technique:* The group as a whole is matched with the other group in terms of Mean and SD of some other variable than the one under study.

Concept and Types of Hypothesis

- An hypothesis is a specific statement of prediction. It describes in concrete terms what a researcher will expect to happen in his or her study.
- Null hypothesis: It specifies that there is no significant difference between specified populations, any observed difference being due to sampling error or experimental error.

Ho: There is no significant difference between the paired scores, any observed difference being due to chances associated with random sampling.

Alternative hypothesis: It specifies that there is significant difference between the two specified population or between the two variables.

Ha: There is significant difference between the paired scores, the observed difference is not due to chance of sampling error or experimental error.

Formula for determining the significant difference between two dependent(correlaated) means

- 1. Formula involving standard error with out calculating correlation (or by difference method when the group size is small i.e n<30)

$$\overline{D} = \frac{\sum D}{n}; \qquad s_D = \sqrt{\frac{\sum (D - \overline{D})^2}{n-1}}; \qquad s_{\overline{D}} = \frac{s_D}{\sqrt{n}};$$

$$t = \frac{\overline{D}}{s_{\overline{D}}}; \qquad df = n - 1.$$

Alternatively, t may be computed directly from the differences (D) between the paired scores and the squared values (D^2) of those differences.

$$t = \frac{\sum D}{\sqrt{\dfrac{n \sum D^2 - (\sum D)^2}{n-1}}}; \qquad df = n - 1.$$

- 2. Formula involving correlation in computing standard error (when the group size is large i.e N>30)

$$s_{\overline{X}_1-\overline{X}_2} = \sqrt{(s_{\overline{X}_1})^2 + (s_{\overline{X}_2})^2 - 2r.s_{\overline{X}_1}.s_{\overline{X}_2}};$$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1-\overline{X}_2}} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{(s_{\overline{X}_1})^2 + (s_{\overline{X}_2})^2 - 2r.s_{\overline{X}_1}.s_{\overline{X}_2}}};$$

$$df = n - 1.$$

Assumptions for t test of paired observations by difference method

- A single group is drawn randomly from the population.
- The two sets of scores of the dependent variable obtained from the single group, first set serves as the scores of control group and second set as that of the experimental group.
- Two sets of scores are correlated with each other.
- The dependent variables whose changes are being studied is a continuous measurement variable.
- The variable has a normal distribution in the population.
- Each paired scores of a sample occurs at random and independent of other paired scores in the sample.
- N is less than 30.

Assumptions of t test for paired observations of matched and large groups

- Two equivalent or matched-pair groups are drawn from the population. The groups are matched with respect to an variable which is either identical or related with the dependent variable.
- The two matched groups are either treated with different levels of independent variables or one of the groups may serve as the control group and other as the experimental group.
- Two sets of scores constitute paired and correlated observations.
- The dependent variables whose changes are being studied is a continuous measurement variable.
- The variable has a normal distribution in the population.
- Each paired scores of a sample occurs at random and independent of other paired scores in the sample.
- N is more than 30.
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Computation of t test for paired observations by difference method(incase of small single group)

The achievement test scores of 10 students, before and after practice, are given below. Does practice make a significant difference in achievement test scores?

Individuals	:	1	2	3	4	5	6	7	8	9	10
Achievement scores	:										
(*i*) before practice	:	72	67	90	97	84	92	65	75	80	69
(*ii*) after practice	:	120	81	110	103	109	137	115	82	110	89

Solution :

The H_0 contends that there is no significant difference between the paired scores, any observed difference being due to mere chances associated with random sampling. To estimate the probability P of the correctness of this H_0, a *two-tail t test by the difference method* is applicable to the paired scores of the small single group.

Table 7.4. Table for t test of achievement test scores by the difference method

Individuals	Achievement test scores		D	D^2
	before practice (X_1)	after practice (X_2)	($X_2 - X_1$)	
1	72	120	+ 48	2304
2	67	81	+ 14	196
3	90	110	+ 20	400
4	97	103	+ 6	36
5	84	109	+ 25	625
6	92	137	+ 45	2025
7	65	115	+ 50	2500
8	75	82	+ 7	49
9	80	110	+ 30	900
10	69	89	+ 20	400
Total			+ 265	9435

(*a*) The paired achievement test scores (X_1 and X_2) of each individual are entered in Table 7.4.

(*b*) The difference D between the two scores of each pair of observations is worked out and recorded in Table 7.4. These differences for all the pairs of scores are totalled to give ΣD.

(*c*) Each such difference is squared to give D^2 which is tabulated. All such D^2 values are totalled to give ΣD^2.

(*d*) The t score is computed, using ΣD, ΣD^2 and the sample size ($n = 10$).

$$t = \frac{\Sigma D}{\sqrt{\dfrac{n\Sigma D^2 - (\Sigma D)^2}{n-1}}} = \frac{265}{\sqrt{\dfrac{10 \times 9435 - (265)^2}{10-1}}} = 5.118; \qquad df = n - 1 = 10 - 1 = 9.$$

(*e*) The computed t is then compared with the *two-tail critical t scores* ($df = 9$) for different levels of significance (Table B of Appendix).

$$t_{.05(9)} = 2.262; \quad t_{.02(9)} = 2.821; \quad t_{.01(9)} = 3.250; \quad t_{.001(9)} = 4.781.$$

As the computed t of 5.118 exceeds even the critical $t_{.001}$, the probability P of correctness of the H_0 is lower than 0.001 and is considered too low. So, the H_0 cannot be retained. Hence, it is inferred that practice produces a significant difference in achievement test scores ($P < 0.001$).

Computation of t test for paired observations of matched and large single groups

Example 7.7.8.

The mean systolic blood pressure of 41 adult women was found to be 166.4 mm Hg (*SE* 8.650) and 160.0 mm Hg (*SE* 7.460) respectively before and after therapy with a vasodilator drug. The correlation coefficient *r* between the initial and final blood pressure scores of all the individuals was found to be + 0.65. Did the mean systolic pressure change significantly on treatment with the drug?

Solution :

The H_0 contends that there is no significant difference between the means. To find the probability *P* of this H_0 being correct, a *two-tail t test* is done using the correlation coefficient between the paired scores of the large single group ($n = 41$).

$$\overline{X}_1 = 166.4 \text{ mm Hg}; \quad s_{\overline{X}_1} = 8.650 \text{ mm Hg};$$
$$\overline{X}_2 = 160.0 \text{ mm Hg}; \quad s_{\overline{X}_2} = 7.460 \text{ mm Hg}; \quad n = 41; \quad r = +0.65.$$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{(s_{\overline{X}_1})^2 + (s_{\overline{X}_2})^2 - 2r.s_{\overline{X}_1}.s_{\overline{X}_2}}} = \frac{166.4 - 160.0}{\sqrt{(8.65)^2 + (7.46)^2 - 2 \times 0.65 \times 8.65 \times 7.46}} = 0.938.$$

$$df = n - 1 = 41 - 1 = 40.$$

Two-tail critical t scores ($df = 40$) are quoted from Table B of Appendix:

$$t_{.05(40)} = 2.021; \quad t_{.02(40)} = 2.423; \quad t_{.01(40)} = 2.704; \quad t_{.001(40)} = 3.551.$$

As the computed t score of 0.938 is lower than even the critical $t_{.05}$, the probability *P* of correctness of H_0 is higher than 0.05 and may be considered too high. So, the H_0 cannot be rejected. It is, therefore, inferred that there is no significant difference between the means ($P > 0.05$). Hence, the mean systolic pressure did not change significantly on treatment with the drug.

Confidence Interval for Mean Difference

- **Confidence interval** is the expected percentage of times that the actual value will fall within the stated precision limit. It is the range of scores within which the parameter has a given probability of lying. To discover whether two groups differ significantly in mean performance we have to say with confidence that there is a difference between the means of the population from which the samples were drawn. To know that first we have to compute the standard error of difference between the two sample means.

  Standard error of the difference between the sample statistics of a particular type is the standard deviation of the sampling distribution of such differences around the difference between the parameters of the population from which the samples have been drawn.
- Then the significance of the difference between the two sample means must be assessed paying due consideration to the SE of that difference. For this the observed difference between the two sample means is generally transformed into a **standard score** i.e either t or Z distribution.
- The Probability of this standard score occurring by mere chance due to random sampling is then found out using either unit normal curve or t distribution. If the probability is found to be too high that the observed difference between the sample means may have arisen from the chances associated with random sampling the observed difference is not considered significant and vise-e-versa.

Relation between confidence interval and hypothesis testing

- In testing the null hypothesis, the mean difference between the two scores is assumed to be zero as it is expected that both the groups belong to the same population. However, if any difference is found it is assumed to be due to the chance of random sampling and expected that the observed differences between the sample means are likely to be fall both above and below the mean difference of zero.
- After calculating CR(critical ratio) value the significant of difference may be evaluated by setting up the confidence intervals for the population difference. Confidence intervals are determined depending on the significance level chosen for the study. The limits specified by D+/-1.96 sdd define the 0.95 confidence interval for the population difference and D +/- 2.58 sdd define the 0.99 confidence interval for the true difference where D means difference score and sdd means SE of the difference score).

- If we take the help of table A we have to find out the percentage of cases fall between the mean +/- CR value. If the percentages of cases in between mean +/- CR value cover either at/less than 95% or 99%(chosen significance level) as decided before depending on the importance of the study, the null hypothesis is accepted and the alternative hypothesis are rejected. (Ref. Garrett-Statistics in Psychology and Education, page number-214-217)

Advantages of confidencee interval

- The confidence interval is the range within which the results may vary but still be acceptable. It indicates the likelihood that the result will fall within that range. Significance level indicates the likelihood that the results will fall outside that range. Confidence level of 95% suggest the significance level of 0.05 and that of 99% suggest the significance level of 0.01. The area of normal curve within precision limits for the specified confidence level constitute the acceptance region and the area of the curve outside the limits in either direction constitutes the rejection regions for null hypothesis. Thus this interval helps us to accept or reject null hypothesis.

References

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Thank You